UNIT-IV

MAGNETO STATICS -II

<u>Magnetic force</u> - Moving charges in magnetic field - Lorentz force equation - force on a Current element - Force on a ~~l~~ straight and ~~I~~ a long current carrying conductor - Force between two straight long and parallel current carrying conductors - Magnetic dipole and dipole moment - a differential ~~loop~~ Current loop as a magnetic dipole - Torque on a current loop.

<u>Magnetic potential</u> : Scalar Magnetic potential and its limitations - Vector magnetic potential and its properties - Vector magnetic potential due to simple configurations - Vector poission's equations.

<u>Inductance</u> - Self and Mutual inductance - determination of Self-inductance of Solenoid and toroid, Neumann's formulae - mutual inductance between a straight long wire and a Square loop wire in the Same plane. Inductance calculation in static magnetic field.

MAGNETIC FORCE

There are at least three ways in which <u>force due to magnetic fields</u> can be experienced.

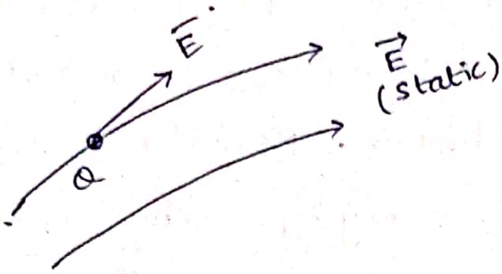
The force Canbe

(a) due to a moving Charged particle in a $\vec{B}$ field

(b) on a Current element in an external $\vec{B}$ field

(c) between two Current elements

(a) FORCE ON A CHARGED PARTICLE :



$+Q \approx$ set free or moving

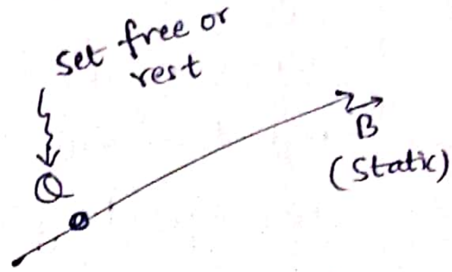
└--> accelerated in the direction of $\vec{E}$ due to electric force $\vec{F_e}$.

According to Coulomb's law

$$\boxed{\vec{F_e} = Q\vec{E}}$$

→ $\vec{F_e}$ does some work.

→ This shows that, if Q is positive, $\vec{F_e}$ and $\vec{E}$ have the same direction



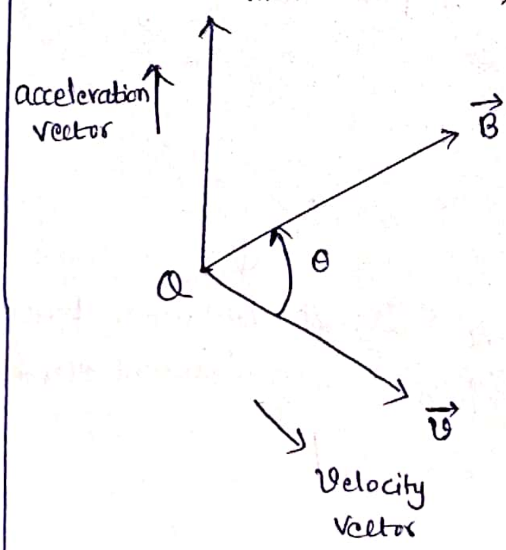
$+Q \approx$ Remains at rest

└--> Nothing Happens

* $\vec{F_m} = 0$ (does no work)

Moving charge in $\vec{B}$:

$$\vec{F_m} = Q(\vec{v} \times \vec{B})$$



$+Q \approx$ move with Velocity $\vec{v}$

└---> experience a force at right angle to both Velocity $\vec{v}$ and $\vec{B}$

Magnitude of $\vec{F_m}$:

$$\boxed{F_m = Q\vec{v}\vec{B}\sin\theta}$$

Case :- Moving Charge in both $\vec{E}$ & $\vec{B}$

Total force $\vec{F} = \vec{F_e} + \vec{F_m}$

$$= Q\vec{E} + Q(\vec{v}\times\vec{B})$$

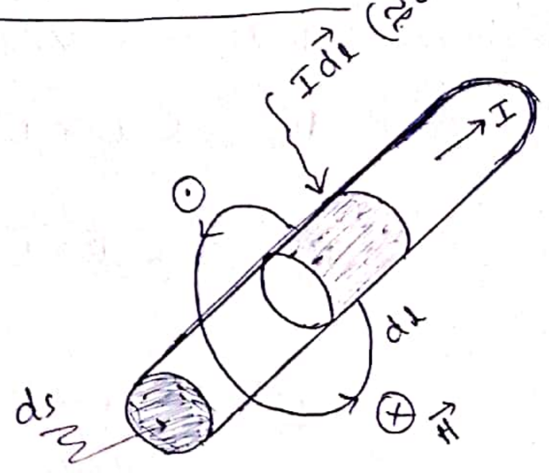
$$\boxed{\vec{F} = Q(\vec{E} + \vec{v}\times\vec{B})}$$

↳ Lorentz force equation.

(8) Table: Force on a charged particle

State of particle	$\vec{E}$ field	$\vec{B}$ field	Combined $\vec{E}$ & $\vec{B}$ Fields
Stationary	$Q\vec{E}$	–	$Q\vec{E}$
Moving	$Q\vec{E}$	$Q\vec{v}\times\vec{B}$	$Q(\vec{E} + \vec{v}\times\vec{B})$

(b) FORCE ON A DIFFENTIAL CURRENT ELEMENT :



$F_m = Q(\vec{v}\times\vec{B})$

& $\vec{J} = \rho_v\cdot\vec{v}$

Current elements :

$$I\vec{dl} \cong \vec{K}\cdot ds \equiv \vec{J}\cdot dv$$

$$\therefore I\vec{dl} = \vec{J}\cdot dv$$

$$= \rho_v\cdot\vec{v}\cdot dv$$

$$= dQ\cdot\vec{v} \quad (\because dQ = \rho_v\cdot dv)$$

Since $dQ = \rho_v\cdot dv$

$dQ = \rho_v\cdot(ds\cdot dl)$

$\Rightarrow dI = \dfrac{dQ}{dt} = \rho_v\cdot ds\cdot\dfrac{dl}{dt} = \rho_v\cdot ds\cdot dv$

$\Rightarrow J = \dfrac{dI}{ds} = \rho_v\cdot dv \Rightarrow \boxed{\vec{J} = \frac{I}{S} = \rho_v\cdot\vec{v}}$

$$\boxed{I\,\vec{dl} = dQ\cdot\vec{v}}$$

→ Elemental charge dQ moving with velocity $\vec{v}$ is equivalent to the conduction current element $I\vec{dl}$.

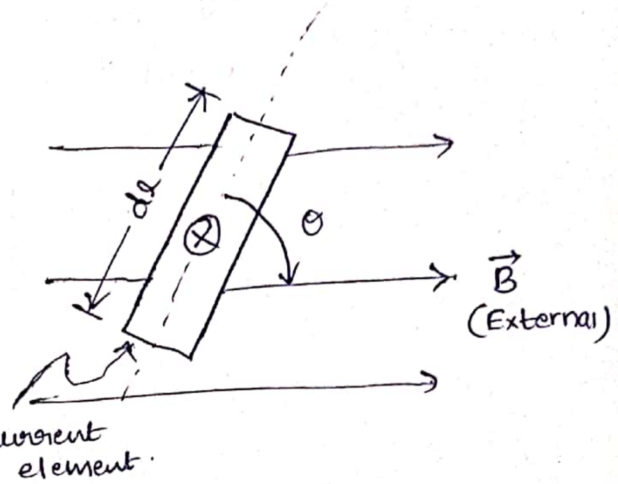
<u>From:</u>

$$\vec{F}_m = Q\,\vec{v}\times\vec{B}$$

$$d\vec{F}_m = dQ\,\vec{v}\times\vec{B}$$

$$d\vec{F}_m = I\,\vec{dl}\times\vec{B}$$

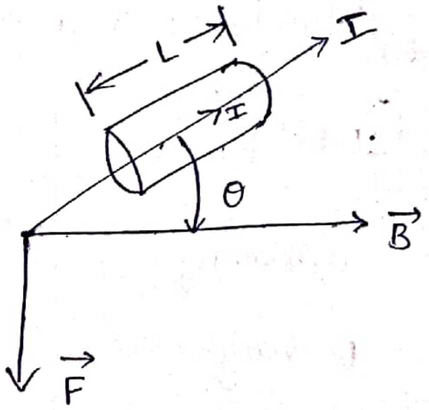
$$\boxed{\vec{F}_m = \oint_L I\vec{dl}\times\vec{B}}$$



<u>In general:</u>

or

$$\boxed{\vec{F}_m = I\,\vec{L}\times\vec{B}}$$



$L \simeq$ Total length of Conductor

$$\boxed{|\vec{F}_m| = F_m = BIL\sin\theta}$$

→ Instead of line current element $I\vec{dl}$, we have surface current elements $\vec{K}ds$ or a volume current element $\vec{J}\cdot dv$

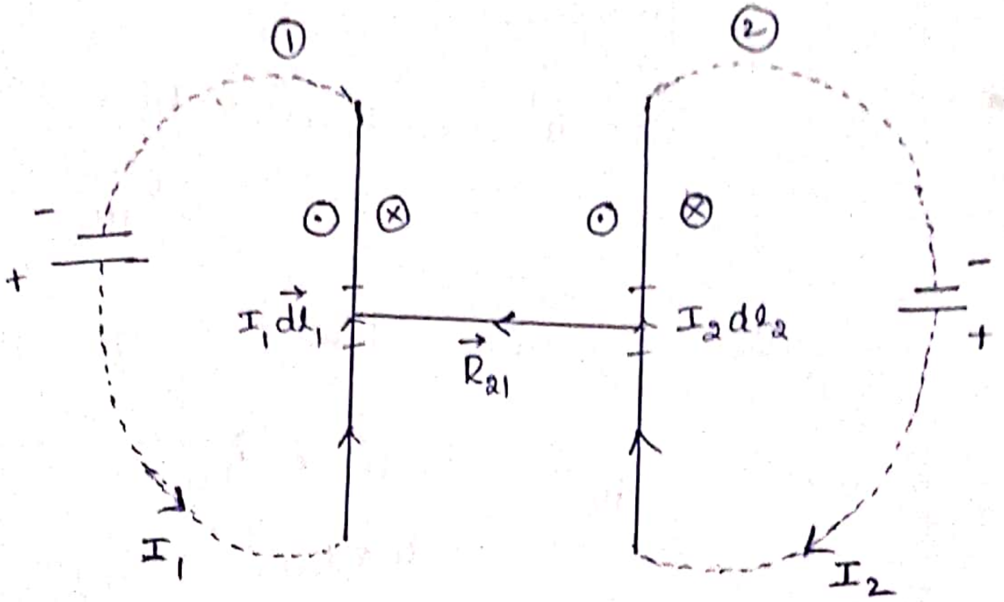
$$d\vec{F}_m = \vec{K}\,ds\times\vec{B} \quad \text{or} \quad d\vec{F}_m = \vec{J}\cdot dv\times\vec{B}$$

$$\vec{F}_m = \int_S \vec{K}\cdot ds\times\vec{B} \quad \text{or} \quad \vec{F}_m = \int_V \vec{J}\cdot dv\times\vec{B}$$

→ $$\boxed{\therefore\ \vec{B} = \frac{\text{Force}}{\text{Current element}}}$$

The magnetic field $\vec{B}$ is defined as the <u>force per unit current element</u>.

(C) FORCE BETWEEN TWO CURRENT ELEMENTS

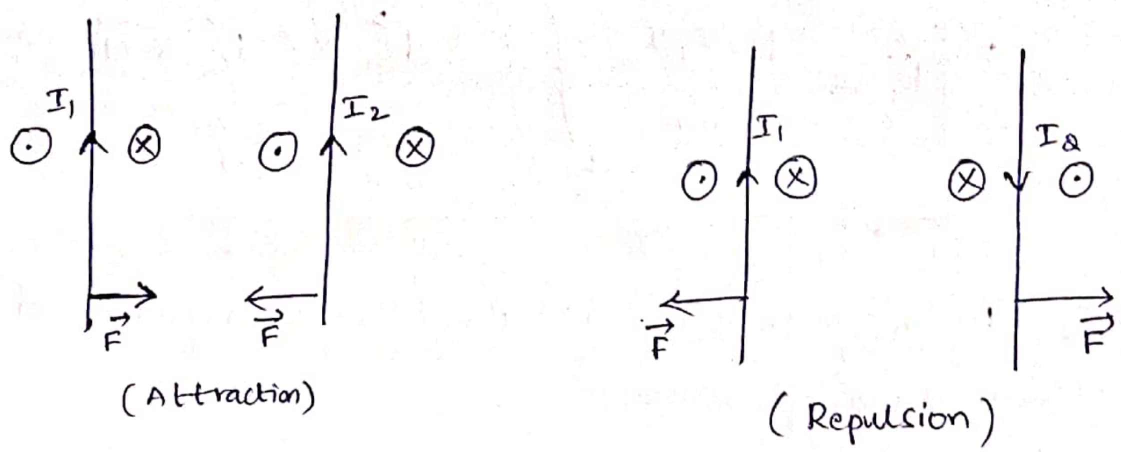


Let us now consider the force between two elements $I_1 dl_1$ & $I_2 dl_2$.

According to Biot-Savart's law, both current elements produce magnetic fields.

Let $d\vec{B}_2$ is the differential magnetic field for loop 2 by the current element $I_2 \vec{dl}_2$.

$d\vec{F}_1$ ≈ differential force on current element $I_1 \vec{dl}_1$ due to $d\vec{B}_1$ i.e., loop 1.



Now, the force on current element $I_1 \vec{dl_1}$ due to $\vec{dB_2}$ is given as

$$d(\vec{dF_1}) = I_1 \vec{dl_1} \times \underbrace{\vec{dB_2}}_{\mu_0 dH_2} \qquad \left[\because \vec{dF} = I\vec{dl} \times \vec{B}\right]$$

Using Biot-Savart's law

$$\vec{dH_2} = \frac{I_2 \vec{dl_2} \times \vec{a}_{R_{21}}}{4\pi R_{21}^2}$$

$$d(\vec{dF_1}) = I_1 \vec{dl_1} \times \left[\frac{\mu_0 \cdot I_2 \vec{dl_2} \times \vec{a}_{R_{21}}}{4\pi R_{21}^2}\right]$$

$$d(\vec{dF_1}) = \frac{\mu_0 I_1 \vec{dl_1} \times I_2 \vec{dl_2} \times \vec{a}_{R_{21}}}{4\pi R_{21}^2}$$

∴ To determine total force, Integral two times $\oint_{L_1}$ & $\oint_{L_2}$

$$\boxed{\vec{F_1} = \frac{\mu_0 I_1 I_2}{4\pi} \oint_{L_1} \oint_{L_2} \frac{\vec{dl_1} \times (\vec{dl_2} \times \vec{a}_{R_{21}})}{R_{21}^2}}$$

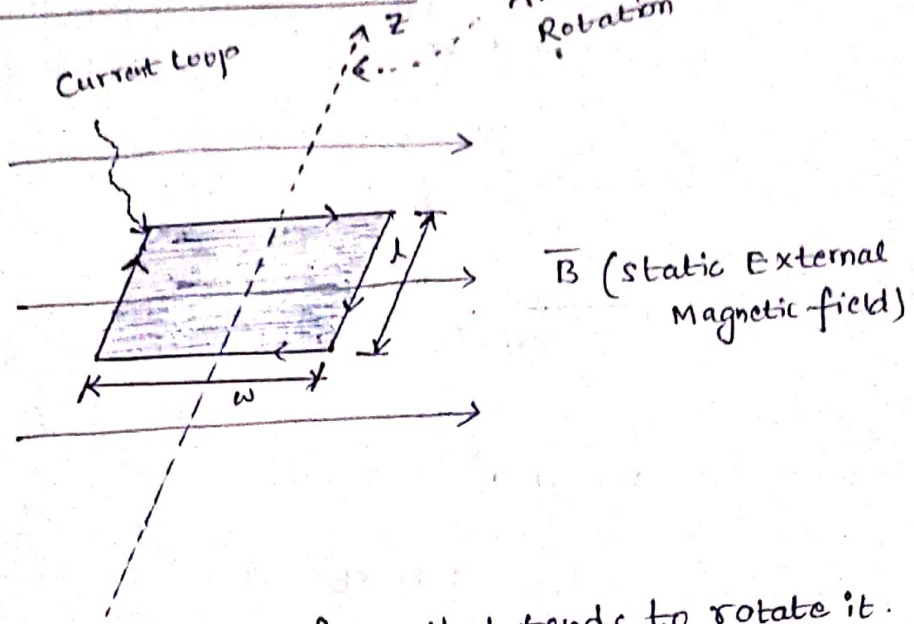
NOTE The magnetic force is attractive or repulsive depends upon the direction of Current.

→ When the Current is flowing in same direction, there is a force of attraction.

→ When the Current is flowing in opposite direction, there is a force of repulsion.

→ For infinite long straight Conductors, $\boxed{F = \frac{\mu_0 I_1 I_2}{2\pi R_{21}}}$

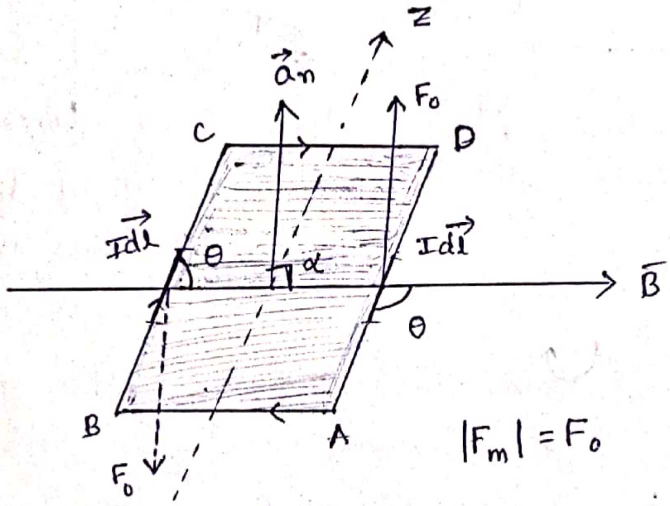
MAGNETIC TORQUE AND MOMENT :



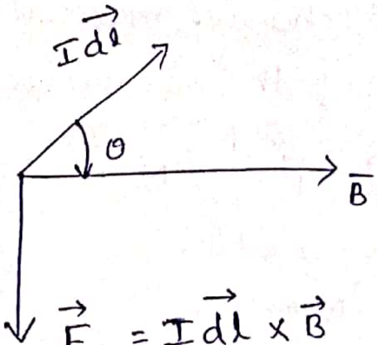
→ Loop experiences a force that tends to rotate it.

≈ Torque in $\vec{B}$

≈ Magnetic Torque

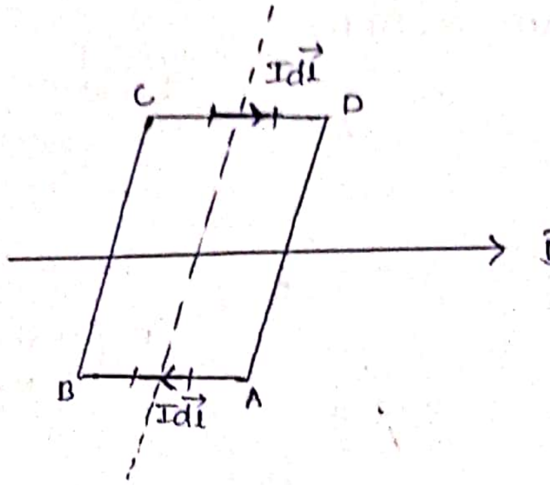


$|F_m| = F_0$



For side BC & DA

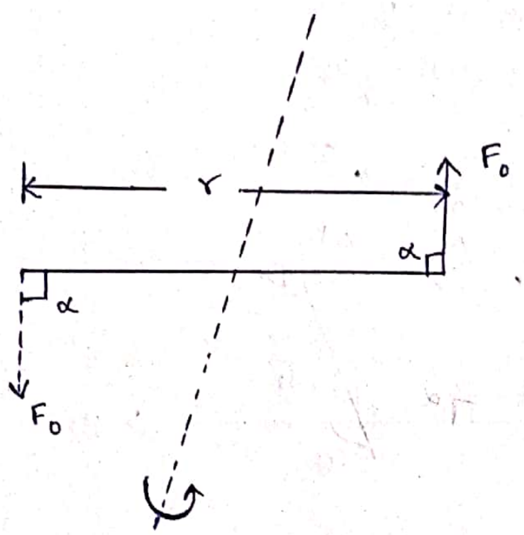
$\vec{F}_m = I\vec{dl} \times \vec{B}$



for side AB & CD

$I\,dl \parallel \vec{B}$, $\theta = 0^\circ$

$\therefore \vec{F}_m = I\,d\vec{l} \times \vec{B} = 0.$



Forces F_0 & $-F_0$ act at different points on the loop creating a Couple.

Magnetic Torque / Magnetic moment of force

The torque (or mechanical moment of force) on the loop is the vector product of the moment arm 'r' and the force 'F'.

$$\boxed{\vec{T} = \vec{r} \times \vec{F}}$$

Force.

N-m

Moment arm

Thus, Total force

$$\vec{F} = I\int_B^C \vec{dl}\times\vec{B} + I\int_D^A \vec{dl}\times\vec{B}$$

$$= I\int_0^l \vec{dl}\times\vec{B} + I\int_l^0 \vec{dl}\times\vec{B}$$

$$= I\left[\int_0^l \vec{dl}\times\vec{B} - \int_0^l \vec{dl}\times\vec{B}\right]$$

$\vec{F} = 0$ .... when Side BC & DA are $\perp$r to $\vec{B}$;

$\theta = 90°$

... Thus no force is exerted on the loop as a whole.

$\because$ F_o is equal and Opposite on side BC & DA.

The Torque,

$$\vec{T} = \vec{r}\times|\vec{F}| = |\vec{r}|\,|\vec{F}|\sin\alpha$$

$$\vec{T} = W \times IBl \times \sin\alpha$$

$$\vec{T} = BIlW \qquad (\because \alpha = 90°)$$

$$\boxed{\vec{T} = BIA} \qquad (\because A = \text{area of loop} = l\times w)$$

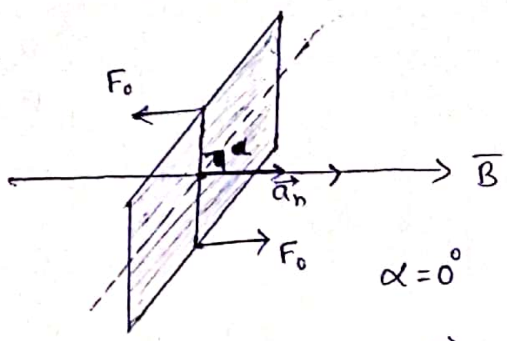
$\hookrightarrow$ (Max Torque)

→ When the Current loop is $\perp$r to $\vec{B}$ vector

$$\vec{T} = \vec{r}\times\vec{F}$$

$$= |\vec{r}|\,|\vec{F}|\sin 0°$$

$$\boxed{\vec{T} = 0}$$



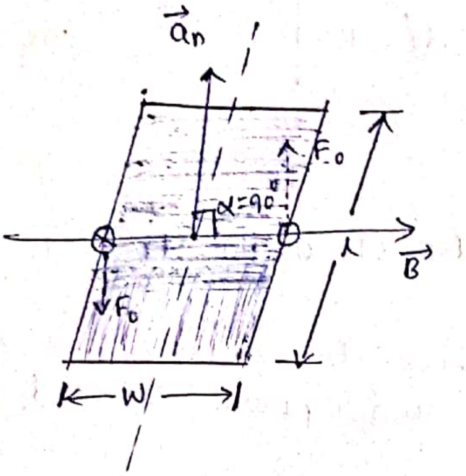
Loop is $\perp$r to $\vec{B}$

NOTE

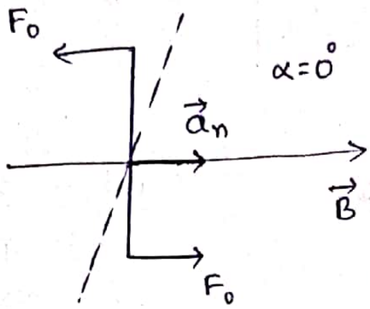
(1) When $\vec{a}_n$ is normal to $\vec{B}$, we get T_{max} $[\because \alpha = 90^\circ]$

(2) When $\vec{a}_n$ is parallel to $\vec{B}$, we get Zero Torque. $[\because \alpha = 0^\circ]$

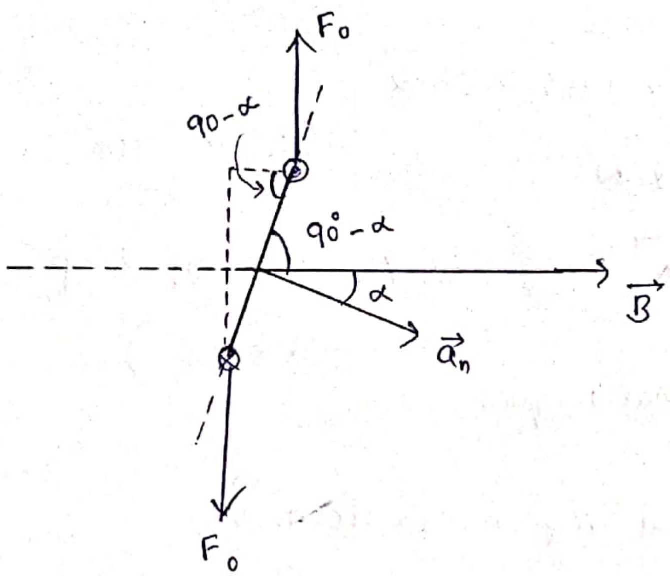
Comparision:



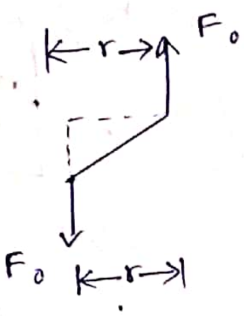
Fig(1): Loop // $\vec{B}$ (T_{max})

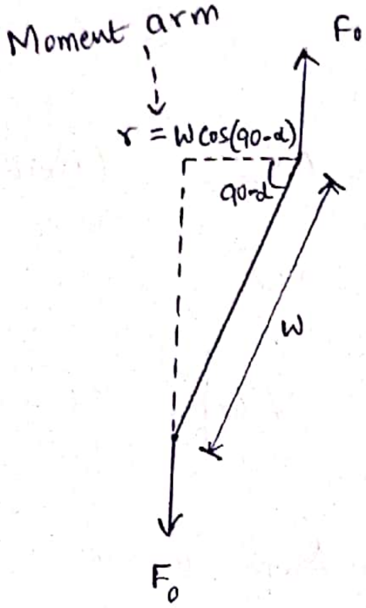


Fig(2): Loop $\perp$r $\vec{B}$ ($T=0$)



Fig(3): loop having $\vec{a}_n$ makes an angle α to $\vec{B}$





$\cos(90-\alpha) = \frac{r}{W}$

$\Rightarrow r = W\cos(90-\alpha)$

$\boxed{r = W\sin\alpha}$

$\vec{T} = \vec{r} \times \vec{F}$

$\vec{T} = BIlW\sin\alpha$

$\vec{T} = BIS\sin\alpha$ $\quad [\because S = l \times W]$ → area of loop

$\boxed{\vec{T} = Bm\sin\alpha}$ where, $m = IS$

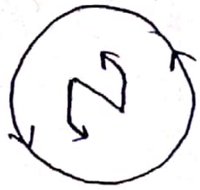
(or) $\quad \vec{m} = IS\,\vec{a}_n$

$\boxed{\vec{T} = \vec{m} \times \vec{B}}$

→ Magnetic dipole moment (A-m²)
= product of Current and area of the loop ; its direction is normal to the loop.

$\boxed{\vec{m} = IS\,\vec{a}_n}$

Current carrying loop behave as a system of two equal and Opposite magnetic forces and Hence is a magnetic dipole.



" product of Current and area of the loop ; its direction is normal to the loop".

MAGNETIC POTENTIAL :

SCALAR MAGNETIC POTENTIAL (V_m) – (units: Amperes)

As in Electrostatic, $\vec{E} = -\nabla V$

So, in Magnetostatic, $\boxed{\vec{H} = -\nabla V_m}$

[$\because \vec{E} \approx \vec{H}$
$\vec{D} \approx \vec{B}$
$\because \nabla \times \vec{E} = 0$]

Since $\nabla \times \vec{H} = \vec{J}$ → Ampere's law

$\nabla \times \vec{H} = \nabla \times (-\nabla V_m)$

$\nabla \times \vec{H} = 0$

$\therefore \vec{J} = 0$

Using Identity

$\nabla \times (\nabla V) = 0$

$\nabla \cdot (\nabla \times \vec{A}) = 0$

NOTE :- Magnetic Scalar potential V_m is only defined in a region where $\vec{J} = 0$

$$\boxed{\vec{H} = -\nabla V_m} \quad \text{if } \vec{J} = 0$$

* The scalar potential satisfies Laplace's equation

$$\boxed{\nabla^2 V_m = 0} \quad (\vec{J} = 0)$$

Since $\nabla \cdot \vec{B} = 0$ ------→ Non-existance of magnetic monopole.

$\mu_0 \nabla \cdot \vec{H} = 0$ $\quad \because \vec{B} = \mu_0 \vec{H}$ .... free space

$\mu_0 \nabla \cdot (-\nabla V_m) = 0$ $\quad \because \vec{H} = -\nabla V_m$

$\boxed{\nabla^2 V_m = 0}$ ..... only for $\vec{J} = 0$

* The scalar magnetic potential directly defined as

$$V_m = -\int_A^B \vec{H} \cdot dl \quad \text{........} \quad (\text{Unit : Ampere})$$

$\vec{H} \approx A/m$

$\vec{dl} \approx m$

VECTOR MAGNETIC POTENTIAL

→ Exist where $\vec{J}$ is present.

" Magnetic Vector potential is defined in such a way that its curl gives the magnetic flux density"

$\boxed{\text{NOTE}}$ (i) When the region having $\vec{J}=0$, there is a scalar magnetic potential.

(ii) When the region having $\vec{J}$, there is a vector magnetic potential.

Using curl because ← $\boxed{\vec{B} = \nabla \times \vec{A}}$ where $\vec{A} \approx$ Vector magnetic potential (Wb/m^2)

Curl of Vector ≈ Vector quantity

Since $\nabla \times \vec{H} = \vec{J}$ .... Ampere's circuit law

$\vec{B} = \mu_0 \vec{H}$ ..... Free space

$\vec{H} = \dfrac{\vec{B}}{\mu_0}$

$\nabla \times \vec{B} = \mu_0 \vec{J}$

Since $\vec{B} = \nabla \times \vec{A}$

$$\nabla \times \vec{B} = \nabla \times \nabla \times \vec{A} = \mu_0 \vec{J}$$

$$\boxed{\begin{array}{l}\text{Using laplacian of Vector} \\ \text{i.e., } \nabla^2 \vec{A} = \nabla(\nabla \cdot \vec{A}) - \nabla \times \nabla \times \vec{A}\end{array}}$$

$$\therefore \underbrace{\nabla(\nabla \cdot \vec{A})}_{=0} - \nabla^2 \vec{A} = \mu_0 \vec{J}$$

Since, For D.C current only $\nabla \cdot \vec{A} = 0$

$$\Rightarrow \quad -\nabla^2 A = \mu_0 \vec{J}$$

(or) $\boxed{\nabla^2 \vec{A} = -\mu_0 \vec{J}}$ ----> Vector poisson's Equation.

$\nabla^2 \vec{A} = -\mu_0 \vec{J}$ .... for (x, y, z) ... c.s

$$\nabla^2 (A_x \bar{a}_x + A_y \bar{a}_y + A_z \bar{a}_z) = -\mu_0 [J_x \bar{a}_x + J_y \bar{a}_y + J_z \bar{a}_z]$$

$$\left.\begin{array}{l} \therefore \nabla^2 A_x = -\mu_0 J_x \\ \nabla^2 A_y = -\mu_0 J_y \\ \nabla^2 A_z = -\mu_0 J_z \end{array}\right\} \text{poisson's equations in Magneto statics.}$$

Just as we defined

$$V = \int \frac{dq}{4\pi\epsilon_0 R}$$

we can define

$$\vec{A} = \int_L \frac{\mu_0 I\, dl}{4\pi R} \quad \text{for line current.}$$

$$\vec{A} = \int_S \frac{\mu_0 K\, ds}{4\pi R} \quad \text{for surface current}$$

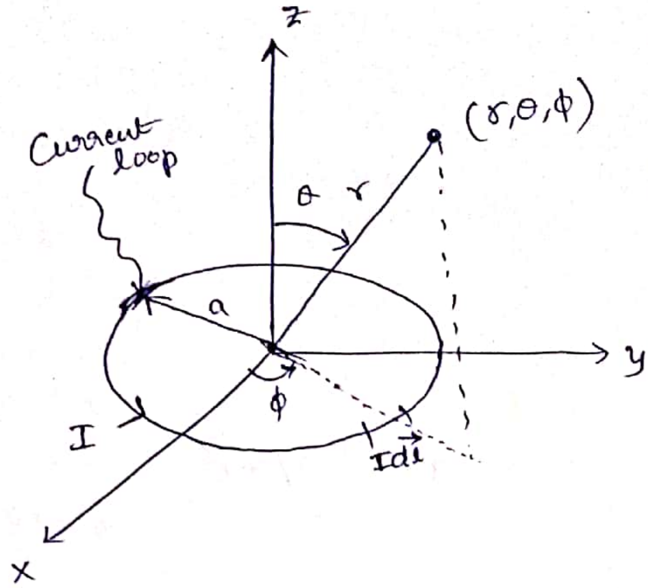
$$\vec{A} = \int_V \frac{\mu_0 J\, dv}{4\pi R} \quad \text{for volume current.}$$

A MAGNETIC DIPOLE

A MAGNETIC DIPOLE

Small current loop. (shown in fig)

Bar magnet



Objective: To determine the magnetic field $\bar{B}$ at point $p(r,\theta,\phi)$ due to circular loop carrying current, I.

Using $\vec{B} = \nabla \times \vec{A}$ where

$\vec{A}$ = Magnetic Vector potential

$$\boxed{\vec{A} = \frac{\mu_0 I}{4\pi} \oint_L \frac{d\vec{l}}{r}}$$

Note: As $r >> a$, loop appear small at point P.

As electric dipole moment has potential

$$V = \frac{\overset{\text{electric dipole moment}}{\vec{m} \cdot \bar{a}_r}}{4\pi\epsilon_0 r^2}$$

V ⟶ scalar

$$\vec{A} = \frac{\mu_0 \cdot \overset{\text{Magnetic dipole moment}}{\vec{m}} \times \vec{a_r}}{4\pi r^2}$$

$\vec{A}$ ⟶ vector

$\because \vec{m} = IS\,\vec{a_n}$

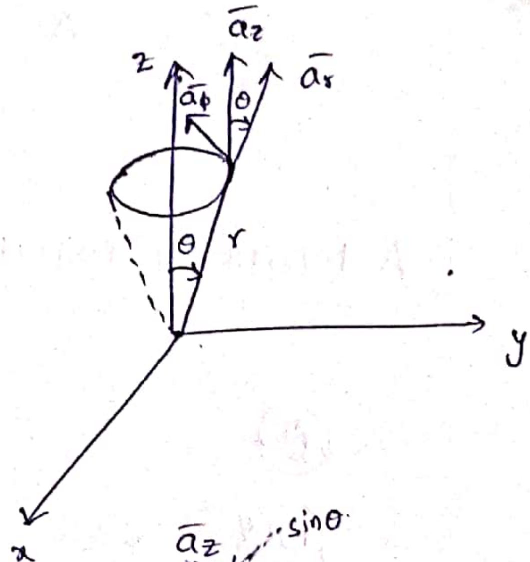
$\vec{m} = I(\pi a^2)\,\bar{a}_z$

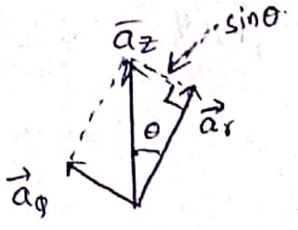
$$\vec{A} = \frac{\mu_0 I (\pi a^2)\, \vec{a_z} \times \vec{a_r}}{4\pi r^2}$$

$$\vec{A} = \frac{\mu_0 I (\pi a^2) \sin\theta \cdot \bar{a}_\phi}{4\pi r^2}$$

$$A_\phi = \frac{\mu_0 I (\pi a^2) \sin\theta}{4\pi r^2}$$

$[\because A_r = A_\theta = 0]$





$\therefore \bar{a}_z \times \bar{a}_r = \sin\theta \cdot \bar{a}_\phi$

$$\vec{B} = \nabla \times \vec{A} = \frac{1}{r^2 \sin\theta} \begin{vmatrix} \bar{a}_r & r\bar{a}_\theta & r\sin\theta\, \vec{a_\phi} \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \phi} \\ 0 & 0 & r\sin\theta \cdot \frac{\mu_0 I (\pi a^2) \sin\theta}{4\pi r^2} \end{vmatrix}$$

$$\bar{B} = \frac{\mu_0 m}{4\pi r^3} (2\cos\theta\, \bar{a}_r + \sin\theta\, \bar{a}_\theta).$$

Inductance - Self Inductance and Mutual Inductance

→ A wire or conductor of certain length, when twisted into coil becomes a basic inductor.

→ For every conductor carrying current I and producing magnetic field $\vec{B}$, there exists a self inductance.

→ When two such coils are placed very close to each other, there exists a mutual inductance between two.

→ A circuit (or closed conducting path) carrying current, I produces a magnetic field $\overline{B}$ that causes a flux $\phi = \int \overline{B} \cdot ds$ to passes through each turn of the circuit as shown in figure.

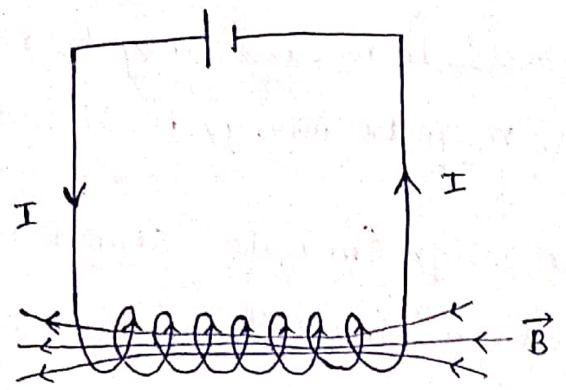


Fig: Magnetic field $\vec{B}$ produced by a circuit.

→ If the circuit has N identical turns, we define the flux linkage λ as

$$\boxed{\lambda = N\phi}$$

Also, if the medium surrounding the circuit is linear, the flux linkage λ is proportional to the current, I producing it;

i.e., $\boxed{\begin{array}{l}\lambda \propto I \\ \lambda = LI\end{array}}$

Where L is the constant of proportionality called the inductance of the circuit.

→ The inductance L is a property of the physical arrangement of the circuit. It is the ability of the physical arrangement to store magnetic energy.

→ A circuit or part of a circuit that has inductance is called an inductor.

So, $$\boxed{L = \frac{\lambda}{I} = \frac{N\phi}{I}}$$ Henry or Webers/Ampere

Self Inductance

Inductance L of an inductor is defined as the ratio of the magnetic flux linkage λ to the current ~~thro~~; I through the inductor.

→ Like capacitances, inductance may be regarded as a measure of how much magnetic energy is stored in an inductor.

The magnetic energy (in joules) stored in an inductor is expressed in circuit theory as

$$W_m = \frac{1}{2} L I^2$$

$$\boxed{L = \frac{2W_m}{I^2}}$$

Thus the self-inductance of a circuit may be defined or calculated from energy considerations.

Instead of having single circuit, we have two circuits carrying current I_1 and I_2 as shown in figure, a magnetic interaction exist between the circuits; Four component fluxes ~~ϕ_{11}, ϕ_{12}, ϕ_{21} and~~ ϕ_{11}, ϕ_{12}, ϕ_{21} and ϕ_{22} are produced.

The flux ϕ_{12} is the flux passing through circuit '1' due to current I_2 in circuit 2.

If B_2 is the magnetic flux density due to I_2 and S_1 is the area of the circuit 1, then

$$\phi_{12} = \int_{S_1} B_2 \cdot ds.$$

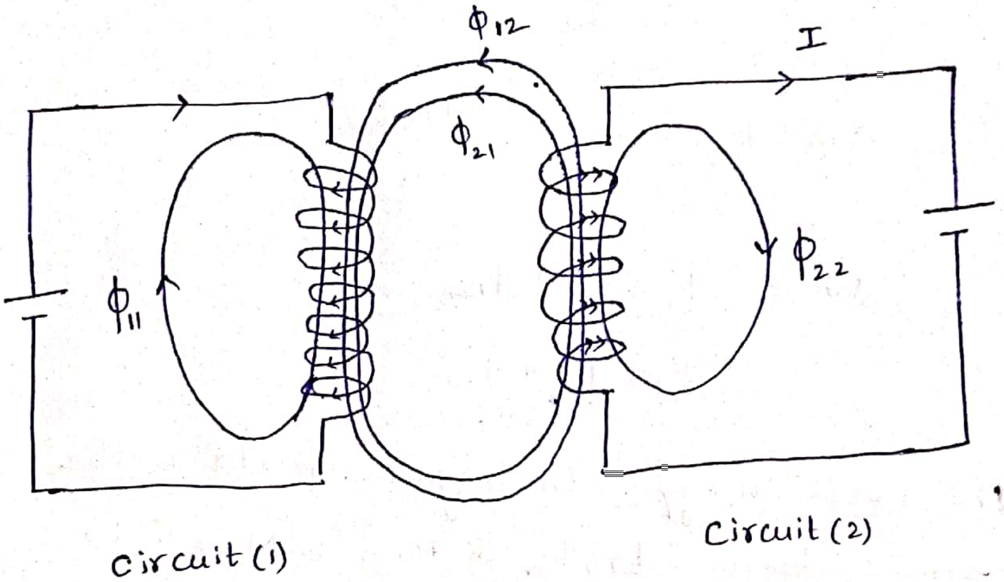


Fig: Magnetic interaction b/w two circuits.

We define the mutual inductance M_{12} as the ratio of the flux linkage $\lambda_{12} = N_1 \phi_{12}$ on circuit 1 to current I_2;

i.e

$$\boxed{M_{12} = \frac{\lambda_{12}}{I_2} = \frac{N_1 \phi_{12}}{I_2}}$$

Similarly, the mutual inductance M_{21} is defined as the flux linkages of Circuit '2' per unit current I_1; i.e.,

$$\boxed{M_{21} = \frac{\lambda_{21}}{I_1} = \frac{N_2 \phi_{21}}{I_1}}$$

The mutual inductance M_{12} or M_{21} is expressed in henrys and should not be confused with the magnetization Vector M expressed in amperes per meter.

Self Inductance of Circuits 1 and 2, respectively, as

$$L_1 = \frac{\lambda_{11}}{I_1} = \frac{N_1 \phi_{11}}{I_1}$$

and
$$L_2 = \frac{\lambda_{22}}{I_2} = \frac{N_2 \phi_{22}}{I_2}$$

where
$$\phi_1 = \phi_{11} + \phi_{12}$$
$$\phi_2 = \phi_{22} + \phi_{21}$$

The total energy in the magnetic field is the sum of the energies due to L_1, L_2 & M_{12} (or M_{21}); i.e.,

$$W_m = W_1 + W_2 + W_3$$
$$= \frac{1}{2} L_1 I_1^2 + \frac{1}{2} L_2 I_2^2 \pm M_{12} I_1 I_2$$

→ The positive sign is taken if currents I_1 and I_2 flow such that the magnetic fields of the two circuits strengthen each other. If the currents flow such that their magnetic fields oppose each other, the negative sign is taken.

Coefficient of Coupling between Two Coils

ϕ_{21} – flux passing through ckt(2) due to Current I_1 in ckt(1)

ϕ_{12} – flux passing through ckt (1) due to Current I_2 in ckt (2)

$$\phi_{21} = K_1 \phi_{11}$$

$$\phi_{12} = K_2 \phi_{22}$$

$$M_{12} = \frac{N_1 \phi_{12}}{I_2} = \frac{N_1 (K_2 \phi_{22})}{I_2}$$

$$M_{21} = \frac{N_2 \phi_{21}}{I_1} = \frac{N_2 (K_1 \phi_{11})}{I_1}$$

Assuming linear medium surrounding two ckts, we can write

$$M_{12} = M_{21} = M$$

$$M^2 = M_{12} \cdot M_{21} = \frac{N_1 (K_2 \phi_{22})}{I_2} \cdot \frac{N_2 (K_1 \phi_{11})}{I_1}$$

$$= K_1 K_2 \cdot \frac{N_1 \phi_{11}}{I_1} \cdot \frac{N_2 \cdot \phi_{22}}{I_2}$$

$$= (K_1 K_2) \cdot L_1 \cdot L_2$$

$$M = \sqrt{K_1 K_2} \cdot \sqrt{L_1 \cdot L_2} \qquad \because K = \sqrt{K_1 K_2}$$

$$\boxed{M = K \cdot \sqrt{L_1 L_2}}$$

Where K is the Coefficient of Coupling between two coils.

$$\boxed{\therefore K = \frac{M}{\sqrt{L_1 L_2}}}$$

Inductance Formulae

(i) Series Aiding : $L = L_1 + L_2 + 2M$

(ii) Series Opposing : $L = L_1 + L_2 - 2M$

(iii) parallel Aiding : $L = \dfrac{L_1 L_2 - M^2}{L_1 + L_2 - 2M}$

(iv) parallel Opposing : $L = \dfrac{L_1 L_2 - M^2}{L_1 + L_2 + 2M}$.

Neuman's Formula :

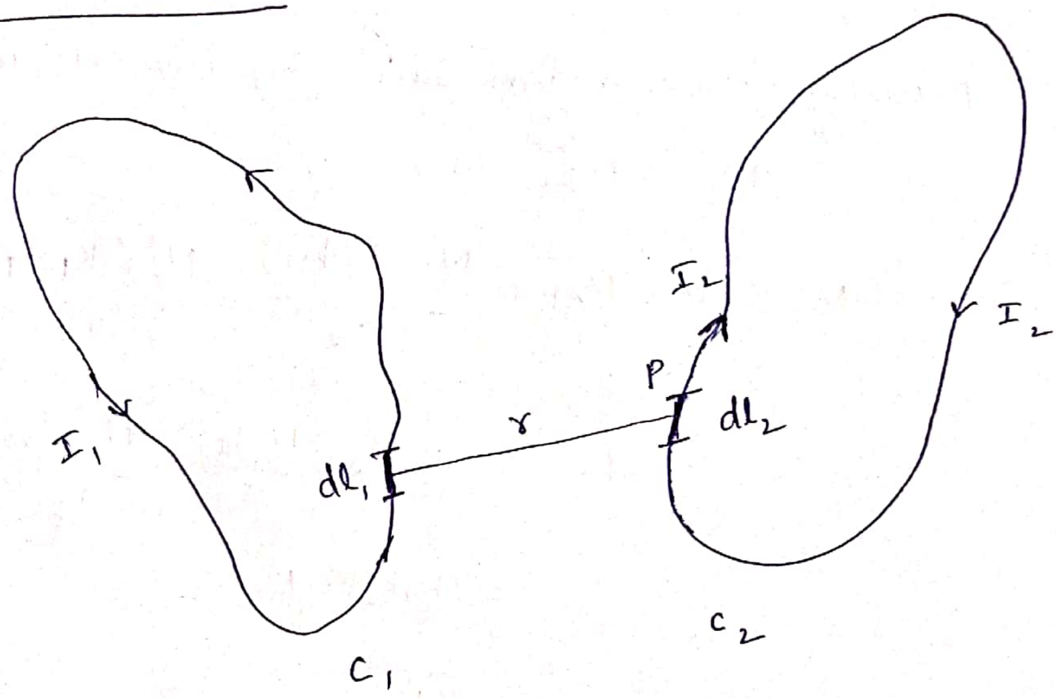


Fig: Two closed paths or loops carrying currents I_1 and I_2.

Consider two closed loops C_1 and C_2 of any random Shape as shown in figure. Both the closed loops are stationary loops placed in a linear medium.

Let I_1 and I_2 be the Currents flowing through closed paths C_1 & C_2 respectively.

Let 'r' be the distance of seperation b/w C_1 and C_2

Let S_1 and S_2 be the Surfaces of loop 1 and loop 2 respectively.

Consider point P located along the surface S_2 of loop 2.

The vector magnetic potential at point P due to loop 1 is given by

$$\vec{A}_1 = \frac{\mu}{4\pi} \oint_{C_1} \frac{I_1 \, dl_1}{r} = \frac{\mu I_1}{4\pi} \oint_{C_1} \frac{\overline{dl_1}}{r}$$

We know

$$\vec{B}_1 = \nabla \times \vec{A}_1$$

Let ϕ_{12} is the flux passing through ckt (1) due to current I_2 in ckt (2).

$$\lambda_{12} = N_1 \phi_{12}$$

Assuming both the ckts of single turn i.e., $N_1 = N_2 = 1$.

Then.

$$\lambda_{12} = \phi_{12} = \int_{S_1} B_2 \cdot ds_1$$

$$\lambda_{21} = \phi_{21} = \int_{S_2} B_1 \cdot ds_2 = \int_{S_2} (\nabla \times \vec{A}_1) \cdot ds_2 .$$

By using stoke's theorem to RHS

$$\lambda_{21} = \oint_{C_2} \vec{A}_1 \cdot \overline{dl_2}$$

$$\lambda_{21} = \oint_{C_2} \left[\frac{\mu}{4\pi} \oint_{C_1} \frac{I_1 \, dl_1}{r} \right] dl_2$$

$$\lambda_{21} = \frac{\mu I_1}{4\pi} \oint_{C_2}\oint_{C_1} \frac{\overline{dl_1} \cdot \overline{dl_2}}{r}$$

But Mutual Inductance

$$M = \frac{\lambda_{21}}{I_1} = \frac{\lambda_{12}}{I_2}$$

$$\boxed{M_{21} = \frac{\mu}{4\pi} \oint_{C_2}\oint_{C_1} \frac{\overline{dl_1} \cdot \overline{dl_2}}{r}}$$

Similarly, we can write

$$\boxed{M_{12} = \frac{\mu}{4\pi} \oint_{C_1}\oint_{C_2} \frac{\overline{dl_1} \cdot \overline{dl_2}}{r}}$$

The above equations are called Neuman's Integrals or Neuman's Formulae.

Inductance of Solenoid

- Consider a Solenoid of N turns as shown in figure.
- Let the Current flowing through the solenoid be I ampere.
- Let the length of the solenoid be 'l' and the cross sectional area be 'A'.

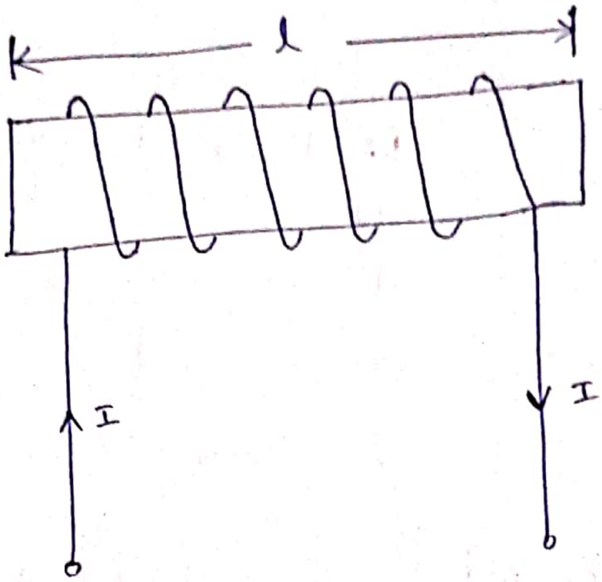


Fig: Solenoid with N turns

- The field intensity inside the solenoid is given by

$$\bar{H} = \frac{NI}{l} \text{ A/m}$$

The total flux linkage is given by,

$$\text{Total flux linkage} = N\phi = NBA = N(\mu H)A$$

$$\text{Total flux linkage} = \mu NHA$$

$$= \mu N\left[\frac{NI}{l}\right]A$$

$$= \frac{\mu N^2 IA}{l}$$

The the inductance of a solenoid is given by

$$L = \frac{\text{Total flux linkage}}{\text{Total current}} = \frac{\mu N^2 \cancel{I} A}{l(\cancel{I})}$$

$$\boxed{L = \frac{\mu N^2 A}{l}} \text{ Henry.}$$

(P) Calculate the inductance of a solenoid of 200 turns wound tightly on a cylindrical tube of 6cm diameter. The length of the tube is 60 cm and the solenoid is in air.

Sol:

For a given solenoid in air,

$$\mu = \mu_0 \mu_r = \mu_0 = 4\pi \times 10^{-7} \ Wb/A\text{-}m$$

$$N = 200$$

$$d = 6cm = 6 \times 10^{-2} m$$

$$\text{hence } r = \frac{d}{2} = 3 \times 10^{-2} m$$

$$l = 60cm = 60 \times 10^{-2} m$$

The inductance of a solenoid is given by

$$L = \frac{\mu N^2 A}{l}$$

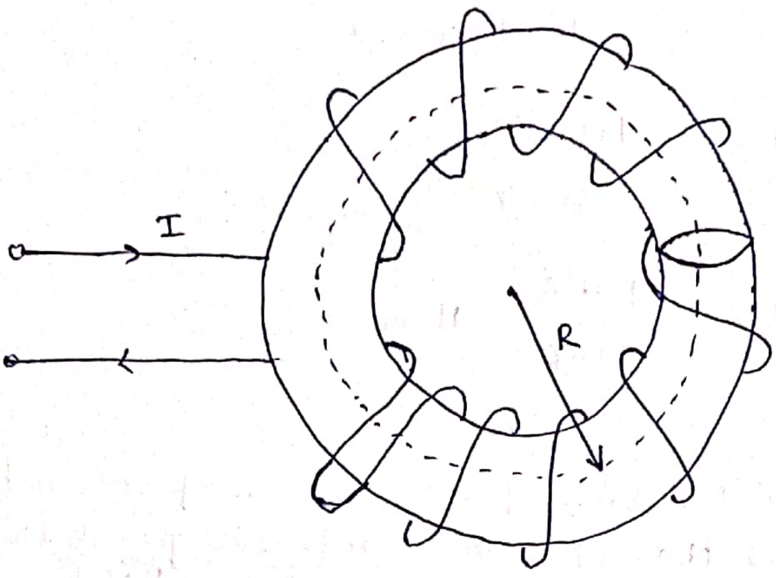
$$= \frac{\mu_0 N^2 (\pi r^2)}{l}$$

$$= \frac{4\pi \times 10^{-7} \times (200)^2 \times \pi \times (3 \times 10^{-2})^2}{60 \times 10^{-2}}$$

$$= 2.3687 \times 10^{-4} H$$

$$= 0.2368 mH$$

Inductance of a Toroid :



(a) Toroidal ring

r : radius of Cross-Section of a ring

Fig(b): Cross-Sectional View of a toroidal ring

→ Consider a toroidal ring with N turns and Carrying Current I. Let the radius of the toroid be R as shown in figure.

The magnetic flux density inside a toroidal ring is given by

$$B = \frac{\mu N I}{2\pi R}$$

The total flux linkage of a toroidal ring having N turns is given by $N\phi$

where $\phi = BA$ [∵ A = Area of Cross-section of a toroidal ring]

∴ Total flux linkage $= N(B)(A)$

$$= N\left[\frac{\mu N I}{2\pi R}\right](A)$$

$$= \frac{\mu N^2 I A}{2\pi R}$$

The inductance of a toroid is given by

$$L = \frac{\text{Total flux linkage}}{\text{Total current}}$$

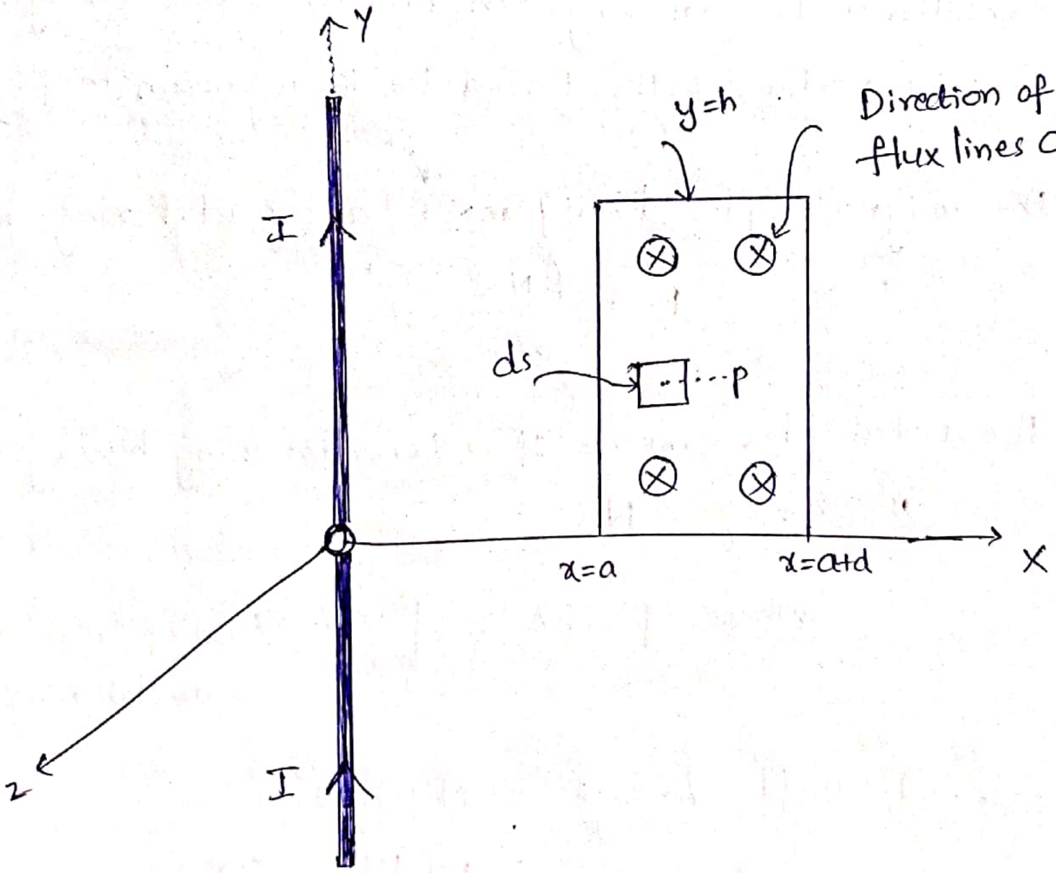
$$L = \frac{\mu N^2 \cancel{I} A}{(2\pi R)(\cancel{I})}$$

$$\boxed{L = \frac{\mu N^2 A}{2\pi R} \text{ Henry}}$$

where A = Area of cross-section of a toroidal ring $= \pi r^2$.

→ For a toroid with N no. of turns, h as the height of toroid with r_1 as inner radius and r_2 as outer radius, $\boxed{L = \frac{\mu N^2 h}{2\pi} \ln\left(\frac{r_2}{r_1}\right)}$ H

→ Mutual Inductance between a straight long wire and Rectangular loop lying in same plane



Let the straight conductor carry a current 'I' in the positive Y-direction.

The flux density at any point 'p' in the XY plane at a distance x from the wire is given by

$$B = \mu_0 \frac{I}{2\pi x}$$

The field produced is in the negative z-direction

→ Mutual Inductance is given by the flux linkage with the loop per unit current in the straight wire.

$$M = \frac{\lambda}{I}$$

$$\lambda = \phi = \iint_S \bar{B} \cdot \hat{n} \cdot ds$$

$$\lambda = \iint_S \mu_0 \frac{I}{2\pi x} \cdot dx \cdot dy \qquad [\because ds = dx \cdot dy$$

$$= \frac{\mu_0 I}{2\pi} \int_{(y=0)}^{h} \int_{(x=a)}^{(a+d)} \frac{1}{x} \, dx \cdot dy .$$

$$\lambda = \frac{\mu_0 I h}{2\pi} \ln\left(\frac{a+d}{a}\right)$$

$$\text{Hence, } M = \frac{\lambda}{I} = \frac{\mu_0 h}{2\pi} \ln\left(1 + \frac{d}{a}\right)$$

→ Instead of rectangular loop, The mutual inductance b/w square loop and a long straight conductor is

$$\boxed{M = \frac{\mu_0 d}{2\pi} \ln\left(1 + \frac{d}{a}\right)}$$

(P) Determine the force per metre length between two straight long parallel wires A and B Seperated by 5 cm in air and carrying currents of 40 amps. (a) in the same direction; (b) in the opposite direction.

Sol: In the expression for force,

$$F = \frac{\mu_0 I_1 I_2}{2\pi D}$$

$$= \frac{2\times10^{-7} I_1 I_2}{D}$$

$$= \frac{2\times10^{-7}(40)(40)}{5\times10^{-2}}$$

$$F = 6.4\times10^{-3} \text{ Newtons}$$

(P) Calculate the inductance of a toroid formed by surfaces $\rho = 3$cm and $\rho = 5$cm, $z=0$ and $z = 1.5$cm wrapped with 5000 turns of wire and filled with a magnetic material with $\mu_r = 6$.

Sol: For toroid,

outer radius $= r_2 = 5\text{cm} = 5\times10^{-2}$ m

inner radius $= r_1 = 3\text{cm} = 3\times10^{-2}$ m

Height, $h = 1.5\text{cm} = 1.5\times10^{-2}$ m

$N = 5000$, $\mu_r = 6$

The inductance of a toroid is given by

$$L = \frac{\mu N^2 h}{2\pi}\ln\left(\frac{r_2}{r_1}\right) = \frac{(4\pi\times10^{-7}\times6)(5000)^2(1.5\times10^{-2})}{2\pi}\ln\left(\frac{5\times10^{-2}}{3\times10^{-2}}\right)$$

$= 0.2298$ H.